

Solution of the Strong CP problem in the low energy effective Standard Model

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Abstract

We consider the low energy effective chiral theory of QCD mesons and the electroweak Goldstone bosons. In this effective theory the pion sector contributes to the gauge boson masses and the Yukawa couplings of the fermions. Consequently the Yukawa sector of quarks and leptons can have a $SU(2)_L \times U(1)_Y \times U(1)_A$ global symmetry even with nonvanishing fermion masses. The extra chiral $U(1)_A$ symmetry can be used to rotate the CP violating $\bar{\theta}G\tilde{G}$ term away which therefore makes no contribution to low energy CP violating effects like the neutron electric dipole moment. The Goldstone mode associated with this $U(1)_A$ symmetry may be identified with the $SU(2)$ singlet meson η_0 .

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The strong CP problem is a naturalness problem [1] arising from the fact that the dimensionless parameter $\bar{\theta}$ in the CP and T odd QCD operator $\bar{\theta} G^{\mu\nu} \tilde{G}_{\mu\nu}$ is constrained [2] by the measurement of the neutron electric dipole moment [3] to be less than 10^{-9} . This parameter receives contributions from various sources like (a) the QCD vacuum angle θ which was invoked by t'Hooft [4] to resolve the $U(1)_A$ problem, (b) diagonalisation of the quark mass matrix involves making an anomalous chiral $U(1)_A$ transformation on the quarks which contributes $\arg \det M_{ij}$ (M_{ij} being the quark mass matrix) to $\bar{\theta}$ and (c) a divergent radiative correction at higher orders in perturbation theory from the CP violating phase in the CKM matrix [5]. The naturalness problem is the problem of explaining, in the absence of a dynamical principle, why these diverse contributions to $\bar{\theta}$ cancel to within 10^{-9} . Peccei and Quinn [6] introduced the idea that a global $U(1)_A$ symmetry may be responsible for the vanishing of $\bar{\theta}$ which was incorporated by introducing an extra Higgs doublet and the associated Goldstone boson, the axion [7], in the standard model. The Peccei-Quinn-Weinberg-Wilczek axion has been experimentally ruled out [8] and so far none of its variants -the invisible axions [9] - have been observed [10], [11] .

We have implemented the Peccei-Quinn idea in a low energy effective theory of the standard model. We consider the effective theory of the QCD mesons and the electroweak Goldstone bosons. The QCD sector has a $U(2)_L \times U(2)_R$ global symmetry which is broken spontaneously to $U(2)_V$ with pions and the η_0 as the Goldstone bosons [12], [13]. The weak interactions of the pions are obtained by gauging the $SU(2)_L \times U(1)_Y$ subgroup of this theory. At low energies the Higgs sector of the electroweak theory is a chiral perturbation theory of the electroweak Goldstone bosons [14]. The coupling of pions π^a and electroweak Goldstone bosons w^a to the W^\pm and Z gauge bosons show that the linear combination $\tilde{w}^a = (v^2 + f^2)^{-1/2}(v w^a + f \pi^a)$ are the unphysical Goldstone bosons which appear as the longitudinal components of the W^\pm and Z bosons (where $f \simeq 92 \text{ Mev}$ and $v = 246 \text{ Gev}$ are the symmetry breaking scales of the QCD and electroweak sectors respectively) [15]. The physical Goldstone bosons -the electroweak pions - are the orthogonal combination $\tilde{\pi}^a = (v^2 + f^2)^{-1/2}(-f w^a + v \pi^a)$. The meson sector also contributes to the Yukawa

couplings of the quarks and leptons. Owing to this additional Yukawa coupling it is possible to impose a $SU(2)_L \times U(1)_Y \times U(1)_A$ global symmetry on the Yukawa sector and still have nonzero masses of the u and d quarks. The extra chiral $U(1)_A$ symmetry can be used to rotate the CP violating $\bar{\theta} G\tilde{G}$ term away while keeping the fermion mass terms invariant. This means that the QCD vacuum angle does not contribute to any low energy CP violating processes like the neutron electric dipole moment. The Goldstone boson associated with the spontaneous breaking of the $U(1)_A$ symmetry is the $SU(2)$ singlet meson the η_0 . The deviations of the low energy effective theory of Goldstone bosons from the standard quark model can be tested experimentally in the rare decays of pions and the η and η' mesons.

The lagrangian of QCD with u and d quarks has a $U(2)_L \times U(2)_R$ global symmetry in the limit of vanishing quark masses. If this symmetry were to hold in the physical spectrum then the low lying hadrons and mesons would appear as parity doublets and since such a parity doubling is not observed in nature it is assumed that the global $U(2)_L \times U(2)_R$ symmetry is broken to $U(2)_V$. The Goldstone bosons arising from this symmetry breaking transform as $U(2)_L \times U(2)_R / U(2)_V \sim SU(2)_A \times U(1)_A$ and are identified with the pseudoscalar pion triplet π^\pm, π^0 and η_0 . The η_0 is considerably heavier than the pions due to the effect of QCD instantons and because the associated $U(1)_A$ symmetry is anomalous [4]. The Goldstone meson matrix is represented by

$$\mathcal{U} = \exp \left(\frac{i2T^a\pi^a}{f} + i\frac{\eta_0}{f}I \right). \quad (1)$$

where T^a are the generators of $SU(2)$ with the normalisation $\text{tr}T^aT^b = (1/2)\delta^{ab}$. The lagrangian can be written as the series

$$\mathcal{L} = \frac{1}{f^2} \partial_\mu \mathcal{U} \partial^\mu \mathcal{U}^\dagger + \dots \quad (2)$$

where the expansion is in powers of $E/4\pi f$ and the breaks down at energy scales $E \sim 4\pi f \sim 1\text{Gev}$. The weak interactions of the pions are obtained by gauging the $SU(2)_L \times U(1)_Y$ subgroup of the global symmetry $SU(2)_L \times SU(2)_R$, the $U(1)_Y$ being generated by the T_R^3 subgroup of $SU(2)_R$. The η_0 is a singlet under $SU(2)_L \times U(1)_Y$. Using the notation

$Q_L = (u_L, d_L)^T$ and $Q_R = (u_R, d_R)^T$, the hypercharge quantum numbers of Q_L and Q_R can be written as $Y_Q = 1/6 + (T^3)_R$. The $SU(3) \times SU(2)_L \times U(1)_Y$ quantum numbers of the chiral fields is displayed in *Table 1*. The meson matrix \mathcal{U} transforms as the quark bilinear $Q_L \bar{Q}_R$. Under $SU(2)_L \times U(1)_Y$ the fields Q_L, Q_R, \mathcal{U} transform as

$$Q_L \rightarrow L Q_L, \quad Q_R \rightarrow Q_R R^\dagger, \quad \mathcal{U} \rightarrow L \mathcal{U} R^\dagger \quad (3a)$$

$$\text{where } L = \exp(i2\alpha_a T_L^a + i2\beta(1/6)) \text{ and } R = \exp(i2\beta(1/6 + T_R^3)). \quad (3b)$$

The covariant derivative acting on \mathcal{U} defined by

$$D_\mu \mathcal{U} = \partial_\mu \mathcal{U} + ig_2 W_\mu^a T^a \mathcal{U} - ig_1 B_\mu \mathcal{U} T_3 \quad (4)$$

transforms under the local chiral transformation (3a,3b) as

$$D_\mu \mathcal{U} \rightarrow L \mathcal{U} R^\dagger \quad (5)$$

and determines the weak couplings of the Golstone mesons. The weak interactions of the pions is given by the lagrangian expansion

$$\mathcal{L} = \frac{1}{f^2} D_\mu \mathcal{U} D^\mu \mathcal{U}^\dagger + V(\mathcal{U}) + \dots \quad (6)$$

The potential $V(\mathcal{U})$ is chosen such that at the minima the chiral field \mathcal{U} has a nonzero vacuum expectation value

$$\langle \mathcal{U} \rangle = \frac{f}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (7)$$

which breaks the symmetry from $SU(2)_L \times U(1)_Y$ to $U_{EM}(1)$ spontaneously. If pions were the only Goldstone bosons in the theory then they would dissappear from the physical spectrum and appear as the longitudinal components of the W^\pm and Z . In the standard model at low energy there will be mixing of the pions with the electroweak Goldstone bosons [15]. To see this we write the electroweak Higgs sector as a chiral lagrangian [14]. The four real fields of the Higgs doublet $\Phi = (\phi^+, \phi^0)^T$ may be represented by a 2×2 matrix valued field

$$\Sigma = \frac{1}{\sqrt{2}}(v + H) \exp\left(\frac{i2w_a T^a}{v}\right) \quad (8)$$

where $v = 246 \text{ GeV}$ is the electroweak symmetry breaking scale, H the physical Higgs field and w^a are the electroweak Goldstone bosons. We shall work in the nonlinear sigma model limit where the mass of H is much larger than the energy scale of the effective theory and we shall drop H from (8). Under $SU(2)_L \times U(1)_Y$ the transformation of Σ is defined as

$$\Sigma \rightarrow L \Sigma R^\dagger \quad (9a)$$

$$\text{where } L = \exp(i2\alpha_a T_L^a + i2\beta \frac{1}{2}(B - L)) \text{ and } R = \exp(i2\beta(T_R^3 + \frac{1}{2}(B - L))). \quad (9b)$$

The hypercharge subgroup is represented by the T_R^3 generator of $SU(2)_R$ and the fermions have hypercharge quantum numbers $Y = (T^3)_R + (1/2)(B - L)$. At the minima of the Higgs potential Σ acquires the vacuum expectation value

$$\langle \Sigma \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (10)$$

and the symmetry of the chiral lagrangian breaks spontaneously from $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ generated by $T_L^3 + T_R^3 + (1/2)(B - L)$. The covariant derivative corresponding to the local gauge transformation (9a,9b) is given by

$$D_\mu \Sigma = \partial_\mu \Sigma + ig_2 W_\mu^a T^a \Sigma - ig_1 B_\mu \Sigma T_3 \quad (11)$$

and transforms under the local transformation (9a,9b) as $D_\mu \Sigma \rightarrow L D_\mu \Sigma R^\dagger$. The gauge interactions of π^a and w^a are obtained by expanding the exponentials in the lagrangian

$$\mathcal{L} = \frac{1}{2} D_\mu \Sigma D^\mu \Sigma^\dagger + \frac{1}{2} D_\mu \mathcal{U} D^\mu \mathcal{U}^\dagger \quad (12)$$

in powers of π^a and w^a . Expanding (12) the terms thus obtained are the Goldstone boson kinetic energy

$$\partial_\mu \pi^a \partial^\mu \pi^a + \partial_\mu \eta_0 \partial^\mu \eta_0 + \partial_\mu w^a \partial^\mu w^a \quad (13)$$

and the gauge boson mass terms

$$\frac{(v^2 + f^2)}{4} \left\{ \frac{g_2^2}{2} W_\mu^a W^{\mu a} + \frac{g_1^2}{2} B_\mu B^\mu - g_1 g_2 B_\mu W_3^\mu \right\} \quad (14)$$

which are diagonalised by $Z = (\cos\theta_W W^3 - \sin\theta_W B)$ and $A = (\sin\theta_W W_3 + \cos\theta_W B)$ with the usual Weinberg angle $\cos\theta_W = g_2^2/(g_1^2 + g_2^2)^{1/2}$ and $\sin\theta_W = g_1^2/(g_1^2 + g_2^2)^{1/2}$ to yield the gauge boson masses

$$M_W = \frac{1}{2} g_2 (v^2 + f^2)^{1/2} \quad (15)$$

and $M_Z = M_W/\cos\theta_W$. We see that the QCD pion mixing contributes an extra $\Delta M_W = (1/2)(f^2/v^2)M_W^0 \simeq 5.71\text{eV}$. This maybe too small to be of experimental significance. Finally the expansion of the lagrangian (12) yields the gauge boson-Goldstone boson interaction term

$$\frac{g_2}{2} W^{\mu a} (v \partial_\mu w^a + f \partial_\mu \pi^a) + \frac{g_1}{2} B^\mu (v \partial_\mu w^3 + f \partial_\mu \pi^3). \quad (16)$$

We see that the linear combination w^a defined as

$$|\tilde{w}^a\rangle = (v^2 + f^2)^{-1/2} \{ v |w^a\rangle + f |\pi^a\rangle \} \quad (17)$$

are the longitudinal components of the W^\pm, Z bosons and can be gauged away from the lagrangian by a $SU(2)_L \times U(1)_Y$ gauge transformation. The Goldstone bosons which remain in the physical spectrum are the combination orthogonal to \tilde{w}^a

$$|\tilde{\pi}^a\rangle = (v^2 + f^2)^{-1/2} \{ v |\pi^a\rangle - f |w^a\rangle \} \quad (18)$$

and may be regarded as the physical pions. Since $f/v \sim 0.36 \times 10^{-3}$ we can see from (18) that the physical pions are largely the QCD pions with a small admixture of electroweak Goldstone bosons.

Turning to the Yukawa sector we shall show that the Yukawa couplings of mesons may be responsible for the resolution of the strong CP problem as it is possible to impose an extra $U(1)_A$ global symmetry and still have nonzero quark masses. Consider the quark sector. The most general $SU(2)_L \times U(1)_Y$ invariant Yukawa couplings of the quarks $Q_L^i = (u_L^i, d_L^i)^T$

and $Q_R^i = (u_R^i, d_R^i)^T$ (where the superscript i is the generation index $u^i = u, c, t$ and $d^i = d, s, b$) is of the form

$$\bar{Q}_L^i \Sigma G^{ij} Q_R^j + \bar{Q}_L^i \mathcal{U} G'^{ij} Q_R^j + h.c \quad (19)$$

Here G^{ij} and G'^{ij} are 2×2 diagonal matrices $G^{ij} = \text{diagonal}(g_u^{ij}, g_d^{ij})$ and $G'^{ij} = \text{diagonal}(g'_u{}^{ij}, g'_d{}^{ij})$. One can perform biunitary transformations on G^{ij} and G'^{ij} to make them real and diagonal in the generation index. The axial $U(1)$ part of such transformations is anomalous and add to the vacuum angle θ term

$$\theta G\tilde{G} \rightarrow (\theta + \arg \det (v G^{ij} + f G'^{ij})) G\tilde{G} \quad (20)$$

It is the coupling $\bar{\theta} \equiv (\theta + \arg \det (v G^{ij} + f G'^{ij}))$ which is restricted by the neutron electric dipole moment to be less than 10^{-9} [3]. In the diagonal basis we have the following expression for the quark masses in terms of the Yukawas defined in (19)

$$m_u = 2^{-1/2} (v g_u + f g'_u) \quad (21a)$$

$$m_d = 2^{-1/2} (v g_d + f g'_d) \quad (21b)$$

(where we have suppressed the generation index). From an analysis of the effect of quark mass on the masses of baryons and mesons [17], the quarks are assigned running masses at 1 Gev of $m_u(1 \text{ Gev}) = 5.1 \pm 1.5 \text{ Mev}$ and $m_d(1 \text{ Gev}) = 8.9 \pm 2.6 \text{ Mev}$.

Now consider a chiral $U(1)_A$ transformation of the quarks

$$Q_L \rightarrow \exp(\frac{-i}{2}A) Q_L \quad , \quad Q_R \rightarrow \exp(\frac{i}{2}A) Q_R \quad (22)$$

where A is a diagonal 2×2 matrix. The meson matrix \mathcal{U} transforms as the quark bilinear $Q_L \bar{Q}_R$ and under (22) it transforms as

$$\mathcal{U} \rightarrow \exp(\frac{-i}{2}A) \mathcal{U} \exp(\frac{-i}{2}A) \quad (23)$$

This symmetry is realised in the Goldstone mode by the transformation

$$\eta_0 I \rightarrow \eta_0 I - f A \quad (24)$$

The meson Yukawa term

$$\bar{Q}_L \mathcal{U} G' Q_R \quad (25)$$

remains invariant under the chiral $U(1)_A$ transformations (22) and (23).

The Σ field is a singlet under (22) and the corresponding quark Yukawa term $\bar{Q}\Sigma Q$ is in general not invariant under (22). The chiral $U(1)_A$ transformation generates a CP odd term from the Σ Yukawa coupling (19) of the form

$$\delta\mathcal{L}_{\bar{C}P} = \frac{v}{\sqrt{2}} \bar{Q} i\gamma_5 \sin(A) G Q. \quad (26)$$

As the chiral transformation (22) is anomalous it has the effect of changing $\bar{\theta}$ by

$$\bar{\theta} \rightarrow \bar{\theta} - \text{tr}(A) \quad (27)$$

Therefore by choosing the quark rotation matrix A such that $\text{tr}(A) = \bar{\theta}$ we can rotate the CP odd gluonic $\bar{\theta}$ term away in favour of the CP odd term (26) in the quark sector. The quark rotation matrix A is further constrained by Dashen's theorem [16] which states that the axial term (26) must be a singlet under $SU(2)_A$ in order that the vacuum be stable against decay into pions. This is achieved by choosing

$$A = \frac{\bar{\theta}}{(g_u + g_d)} \begin{pmatrix} g_d & 0 \\ 0 & g_u \end{pmatrix} \quad (28)$$

which reduces the CP odd contribution of the Yukawa sector to the flavour singlet form

$$\delta\mathcal{L}_{\bar{C}P} = \frac{v}{\sqrt{2}} \sin\left(\bar{\theta} \frac{g_u g_d}{g_u + g_d}\right) \{ \bar{u} i\gamma_5 u + \bar{d} i\gamma_5 d \} \quad (29)$$

Therefore if the Yukawa couplings g_u or g_d or both are zero then quark Yukawa sector has an additional $U(1)_A$ symmetry which can be used to rotate the gluonic $\bar{\theta}$ term away without generating CP violation in the quark sector. This solves the strong CP problem even with nonzero quark masses as the Yukawa couplings g'_u and g'_d in (21a,21b) can be chosen nonzero.

The $U(1)_A$ symmetry is realised in the Goldstone mode and the corresponding Goldstone boson η_0 can be identified with the flavour singlet linear combination of the $\eta'(958)$ and the $\eta(549)$ mesons. This meson is heavier than other mesons in the (π, K, η_8) octet due to the anomaly in the corresponding $U(1)_A$ symmetry and the effect of QCD instantons [4]. The η_0 mass is given by

$$(m_{\eta_0})^2 = \frac{1}{f} \langle 0 | \frac{3\alpha_s}{8\pi} G\tilde{G} + \sum_i m_i \bar{q}_i i\gamma_5 q_i | \eta_0 \rangle \quad (30)$$

and it does not vanish in the chiral limit of vanishing quark masses unlike the masses of the other Goldstone mesons.

Leptons can also have Yukawa coupling with \mathcal{U} . We write the lepton fields in the chiral notation $L_L^i = (\nu_L^i, e_L^i)^T$ and $L_R^i = (\nu_R^i, e_R^i)^T$ where $i = e, \mu, \tau$ is the generation index. Here we have introduced a right handed neutrino and in the minimal standard model, terms in the chiral expansion involving ν_R may be dropped. The $SU(2)_L \times U(1)_Y$ invariant lepton Yukawa terms are

$$\bar{L}_L \Sigma G_l L_R + \bar{L}_L \mathcal{U} G'_l L_R + \text{h.c} \quad (31)$$

where $G_l = \text{diagonal}(g_\nu, g_e)$ and $G'_l = \text{diagonal}(g'_\nu, g'_e)$ and we have dropped the generation index. The neutral and charged lepton masses are in terms of the Yukawas

$$m_\nu = 2^{-1/2} (v g_\nu + f g'_\nu) \quad (32a)$$

$$m_e = 2^{-1/2} (v g_e + f g'_e). \quad (32b)$$

When we rotate away the $\bar{\theta}$ term by the $U(1)_A$ transformation (22) and (23) generated by (28), the \mathcal{U} Yukawa coupling in (31) gives rise to the CP odd terms

$$\mathcal{L}_{CP} = \frac{\bar{\theta} f}{\sqrt{2}(g_u + g_d)} \{g_d g'_\nu (\bar{\nu} i\gamma_5 \nu) + g_u g'_e (\bar{e} i\gamma_5 e)\} \quad (33)$$

This term which would contribute to the electric dipole moments of neutrino and electron could be made to vanish by choosing $g_d g'_\nu = g_u g'_e = 0$ and thereby imposing the chiral $U(1)_A$ symmetry on the lepton Yukawa sector also.

A probe into the Yukawas are the rare leptonic decay of pions and η_0 . The weak couplings of pions with leptons are obtained in different ways in the quark composite and the Goldstone boson model of pions. From (16) it is clear that the weak vector bosons W^a couple to \tilde{w}^a which are orthogonal to the physical pions $\tilde{\pi}^a$ given by (18). This leads to the paradoxical result that the weak current

$$J_{\mu}^a{}_{weak} = v\partial_{\mu}w^a + f\partial_{\mu}\pi^a \quad (34)$$

does not couple to the physical pions

$$\langle 0 | J_{\mu}^a{}_{weak} | \tilde{\pi}^a \rangle = 0 \quad (35)$$

in the Goldstone bosons model of pions.

We shall see that the weak leptonic decays of the pions for example $\tilde{\pi}^+ \rightarrow \mu^+ \nu$ arise from the Yukawa couplings of the pions with leptons. We expand the Yukawa term (31) in terms of π^a and w^a and use the relations (17) and (18) to substitute in terms of the physical pions $\tilde{\pi}^a$ and the electroweak Goldstone bosons \tilde{w}^a . The \tilde{w}^a can be transformed away by a $SU(2)_L \times U(1)_Y$ gauge transformation. The couplings of $\tilde{\pi}^a$ with fermions are in the leading order

$$\begin{aligned} \mathcal{L}_{\tilde{\pi}ff} = & \frac{i\sqrt{2}}{(v^2 + f^2)} \left\{ \frac{1}{\sqrt{2}}(fg_{\nu} + v)g'_{\nu} \tilde{\pi}^0 \bar{\nu}_L \tilde{\nu}_R - \frac{1}{\sqrt{2}}(fg_l + vg'_l) \tilde{\pi}^0 \bar{e}_L e_R \right. \\ & \left. + (fg_l + vg'_l)V_{ud} \tilde{\pi}^+ \bar{\nu}_L e_R + (fg_{\nu} + vg'_{\nu})V_{du} \tilde{\pi}^- \bar{e}_L \nu_R \right\} + h.c \end{aligned} \quad (36)$$

Here V_{ij} are the Kobayashi-Masakawa matrix elements introduced in going from the weak quark basis to the mass basis during the diagonalisation of (19) and the generation index of leptons has been dropped. The amplitude for the standard weak decay $\pi^+ \rightarrow \mu_R^+ \nu_L$ obtained from (36) is

$$\mathcal{M}_{\tilde{\pi}^+ \rightarrow \mu_R^+ \nu_L} = i\sqrt{2} \left(\frac{fm_{\mu}}{v^2} + vg'_{\mu} \right) V_{ud} \tilde{\pi}^+ \mu_R^+ \nu_L \quad (37)$$

where the second term gives the deviation from the standard quark model. The amplitude for decay in the wrong helicity channel is

$$\mathcal{M}_{\tilde{\pi}^+ \rightarrow \mu_L^+ \nu_R} = i\sqrt{2}(\frac{f m_\nu}{v^2} + v g'_\nu) V_{ud} \tilde{\pi}^+ \mu_L^+ \nu_R \quad (38)$$

Similarly the amplitudes for the decays $\tilde{\pi}^0 \rightarrow \nu \bar{\nu}$, $e^+ e^-$ can be read off from (36) to be compared with the experimentally observed branching ratios.

The couplings of the η_0 to fermions is given by the $\bar{L}_L \mathcal{U} G'_l L_R$ term of the lepton Yukawa coupling (31). The amplitudes for η_0 decay to charged and neutral lepton pairs is given by

$$\mathcal{M}_{\eta_0 \rightarrow e^+ e^-} = (\frac{f}{\sqrt{2}} g'_e) \eta_0 \bar{e} i \gamma_5 e \quad (39a)$$

$$\mathcal{M}_{\eta_0 \rightarrow \nu \bar{\nu}} = (\frac{f}{\sqrt{2}} g'_\nu) \eta_0 \bar{\nu} i \gamma_5 \nu \quad (39b)$$

The simplest ansatz for imposing a chiral $U(1)_A$ symmetry on the Yukawas and avoiding tree level leptonic decays of η_0 would be to let the quark masses be generated entirely from the coupling with \mathcal{U} (i.e set $G = 0$ in (19)) and the leptons masses from Σ (set $G'_l = 0$ in (31)). This however is not the only possibility and the Yukawas must be determined from experiment.

ACKNOWLEDGMENTS

I thank Rabi Mohapatra, Goran Senjanovic and Charan Aulakh for very helpful discussions.

REFERENCES

- [1] R. Peccei in *CP Violation* Ed. C. Jarlskog, World Scientific, Singapore 1989.
- [2] V. Baluni, Phys Rev D 19 (1979) 2227; R. Crewther, P Di Vecchia, G. Veneziano and E. Witten , Phys Lett 88B (1979) 123.
- [3] I.S. Altarev et al JETPL 44 (1986)460; J.M .Pendlebury et al Phys Letters B136(1984) 327.
- [4] G. t'Hooft Phys Rev Letters ,37 (1976) 8; *ibid* Phys Rev D14 (1976) 3432.
- [5] J. Ellis and M.K. Gaillard , Nucl Phys B150 (1979)141.
- [6] R. Peccei and H. Quinn , Phys Rev Letters , 38 (1977)1440 ; *ibid* Phys Rev D16 (1977) 1791.
- [7] S. Weinberg, Phys Rev Letters 40 (1978) 223; F. Wilczek , Phys Rev Letters ,40 (1978) 271 .
- [8] Y. Asano et al Phys. Lett. 107B (1981) 159.
- [9] J.E Kim , Phys Rev Lett 43 (1979) 103 ; M.A.Shifman, A.I.Vainshtein and V.I. Zakarov, Nucl Phys B 166 (1980) 493; M.Dine , W. Fischler and M. Srednicki , Phys. Lett 104 B(1981) 199; A.P. Zhinitski , Sov. J. Nucl Phys 31 (1980) 260.
- [10] J.E. Kim , Phys Rep. 150 (1987) 159.
- [11] S. Mohanty and S.N.Nayak, Phys Rev Lett. 70 (1993) 4038.
- [12] H. Georgi, *Weak interactions and modern particle theory*, Benjamin/Cummings , 1984.
- [13] J. Donoghue, E. Golowich and B. Holstein, *Dynamics of the Standard Model* , Cambridge University Press (1992).
- [14] T. Appelquist and C. Bernard , Phys.Rev D36 (1980)200 ; A. Longhitano , Nucl Phys B188 (1981) 118.

- [15] E. Farhi , *TASI Lectures 1991*, World Scientific, Singapore ,1992.
- [16] R. Dashen, Phys Rev 183 (1969) 1245.
- [17] J.Gasser and H. Leutwyler, Phys Rep 87 (1982) 77.

TABLES

TABLE I. $SU(3) \times SU(2)_L \times U(1)_Y$ quantum numbers of the chiral fields.

Field	$SU(3) \times SU(2)_L \times U(1)_Y$
$Q^i_L = (u^i_L, d^i_L)^T$	$(3, 2, 1/6)$
$Q^i_R = (u^i_R, d^i_R)^T$	$(3, 2, 1/6 + T^3_R)$
$L^i_L = (\nu^i_L, e^i_L)^T$	$(1, 2, -1/2)$
$L^i_R = (\nu^i_R, e^i_R)^T$	$(1, 1, -1/2 + T^3_R)$
Σ	$(1, 2, -T^3_R)$
\mathcal{U}	$(1, 2, -T^3_R)$